Connecting the Concepts

We can read information about the graph of a quadratic function directly from the constants a, h, and k in $f(x) = a(x - h)^2 + k$.

a: Determines the width of the parabola
Tells whether the graph opens up or down
Tells whether the function has a maximum or a minimum

$$f(x) = a(x - h)^2 + k$$

h: Gives the axis of symmetry of the graph

k: Gives the maximum or minimum function value

h and k: Together, give the vertex of the graph

8.6 Exercise Set

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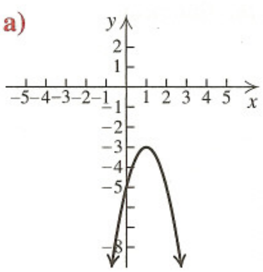
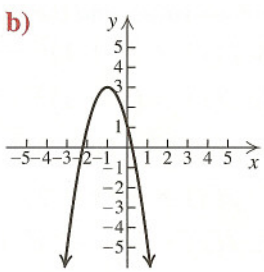
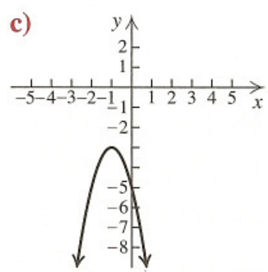
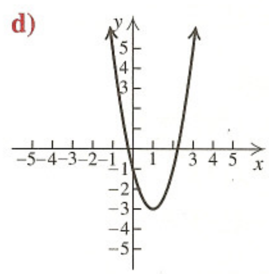
Concept Reinforcement *In each of Exercises 1–8, match the equation with the corresponding graph from those shown.*

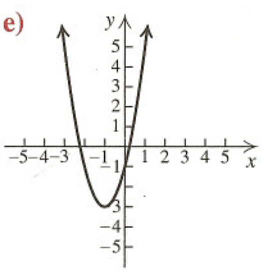
1. (h) $f(x) = 2(x - 1)^2 + 3$
2. (g) $f(x) = -2(x - 1)^2 + 3$
3. (f) $f(x) = 2(x + 1)^2 + 3$
4. (d) $f(x) = 2(x - 1)^2 - 3$
5. (b) $f(x) = -2(x + 1)^2 + 3$
6. (c) $f(x) = -2(x + 1)^2 - 3$
7. (e) $f(x) = 2(x + 1)^2 - 3$
8. (a) $f(x) = -2(x - 1)^2 - 3$

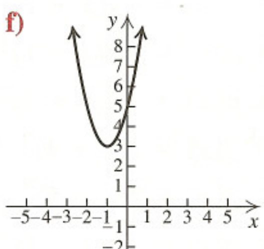
a)


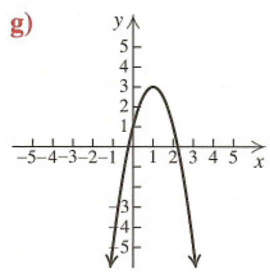
b)


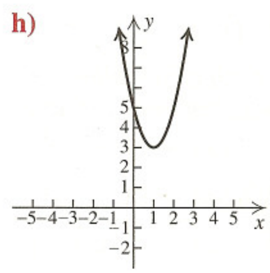
c)


d)


e)


f)


g)


h)


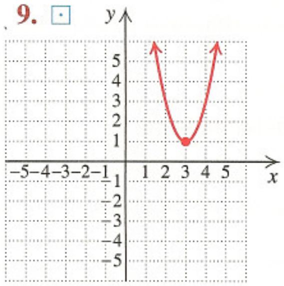
For each graph of a quadratic function

$$f(x) = a(x - h)^2 + k$$

in Exercises 9–14:

a) *Tell whether a is positive or negative.*
b) *Determine the vertex.*
c) *Determine the axis of symmetry.*
d) *Determine the range.*

9.



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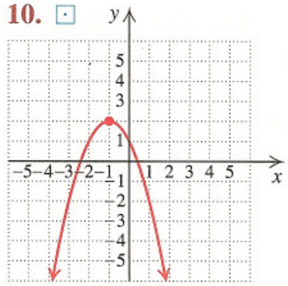

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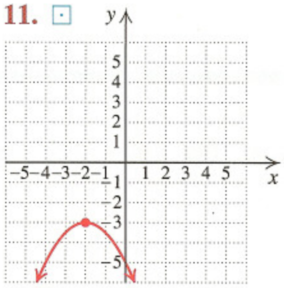

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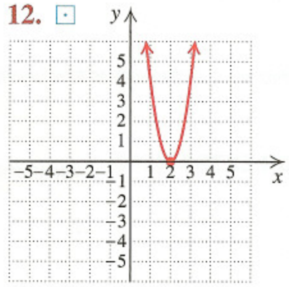
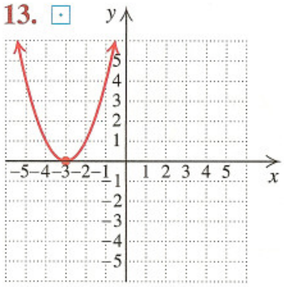
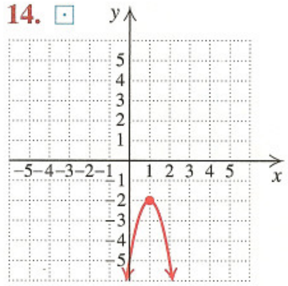


13.



14.



Graph.

15. $f(x) = x^2$

16. $f(x) = -x^2$

17. $f(x) = -2x^2$

18. $f(x) = -3x^2$

19. $g(x) = \frac{1}{3}x^2$

20. $g(x) = \frac{1}{4}x^2$

Aha! **21.** $h(x) = -\frac{1}{3}x^2$

22. $h(x) = -\frac{1}{4}x^2$

23. $f(x) = \frac{5}{2}x^2$

24. $f(x) = \frac{3}{2}x^2$

For each of the following, graph the function, label the vertex, and draw the axis of symmetry.

25. $g(x) = (x + 1)^2$

26. $g(x) = (x + 4)^2$

27. $f(x) = (x - 2)^2$

28. $f(x) = (x - 1)^2$

29. $f(x) = -(x + 1)^2$

30. $f(x) = -(x - 1)^2$

31. $g(x) = -(x - 2)^2$

32. $g(x) = -(x + 4)^2$

33. $f(x) = 2(x + 1)^2$

34. $f(x) = 2(x + 4)^2$

35. $g(x) = 3(x - 4)^2$

36. $g(x) = 3(x - 5)^2$

37. $h(x) = -\frac{1}{2}(x - 4)^2$

38. $h(x) = -\frac{3}{2}(x - 2)^2$

39. $f(x) = \frac{1}{2}(x - 1)^2$

40. $f(x) = \frac{1}{3}(x + 2)^2$

41. $f(x) = -2(x + 5)^2$

42. $f(x) = 2(x + 7)^2$

43. $h(x) = -3\left(x - \frac{1}{2}\right)^2$

44. $h(x) = -2\left(x + \frac{1}{2}\right)^2$

For each of the following, graph the function and find the maximum value or the minimum value and the range of the function.

45. $f(x) = (x - 5)^2 + 2$

46. $f(x) = (x + 3)^2 - 2$

47. $f(x) = -(x + 2)^2 - 1$

48. $f(x) = -(x - 1)^2 + 3$

49. $g(x) = \frac{1}{2}(x + 4)^2 + 3$

50. $g(x) = 2(x - 4)^2 - 1$

51. $h(x) = -2(x - 1)^2 - 3$

52. $h(x) = -\frac{1}{2}(x + 2)^2 + 1$

For each of the following, graph the function and find the vertex, the axis of symmetry, the maximum value or the minimum value, and the range of the function.

53. $f(x) = (x + 1)^2 - 3$

54. $f(x) = (x - 1)^2 + 2$

55. $g(x) = -(x + 3)^2 + 5$

56. $g(x) = -(x - 2)^2 - 4$

57. $f(x) = \frac{1}{2}(x - 2)^2 + 1$

58. $f(x) = -\frac{1}{2}(x + 1)^2 - 1$

59. $h(x) = -2(x - 1)^2 - 3$

60. $h(x) = -2(x + 1)^2 + 4$

61. $f(x) = 2(x + 4)^2 + 1$

62. $f(x) = 2(x - 5)^2 - 3$

63. $g(x) = -\frac{3}{2}(x - 1)^2 + 4$

64. $g(x) = \frac{3}{2}(x + 2)^2 - 3$

Answers to Exercises 9–64 are on pp. IA-18 and IA-19.

Without graphing, find the vertex, the axis of symmetry, and the maximum value or the minimum value.

65. $f(x) = 6(x - 8)^2 + 7$ Vertex: $(8, 7)$; axis of symmetry: $x = 8$; minimum: 7

66. $f(x) = 4(x + 5)^2 - 6$ Vertex: $(-5, -6)$; axis of symmetry: $x = -5$; minimum: -6

67. $h(x) = -\frac{2}{7}(x + 6)^2 + 11$ ⊡

68. $h(x) = -\frac{3}{11}(x - 7)^2 - 9$ ⊡

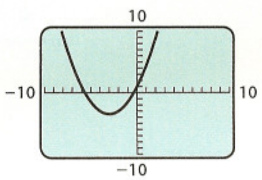
69. $f(x) = \left(x - \frac{7}{2}\right)^2 - \frac{29}{4}$ ⊡

70. $f(x) = -\left(x + \frac{3}{4}\right)^2 + \frac{17}{16}$ ⊡

71. $f(x) = \sqrt{2}(x + 4.58)^2 + 65\pi$ ⊡

72. $f(x) = 4\pi(x - 38.2)^2 - \sqrt{34}$ Vertex: $(38.2, -\sqrt{34})$; axis of symmetry: $x = 38.2$; minimum: $-\sqrt{34}$

TW **73.** While trying to graph $y = -\frac{1}{2}x^2 + 3x + 1$, Ibrahim gets the following screen. How can Ibrahim tell at a glance that a mistake has been made?



TW **74.** Explain, without plotting points, why the graph of $y = (x + 2)^2$ looks like the graph of $y = x^2$ translated 2 units to the left.

SKILL REVIEW

To prepare for Section 8.7, review finding intercepts and completing the square (Sections 2.3, 5.4, 5.5, and 8.1).

Find the x-intercept and the y-intercept. [2.3]

75. $8x - 6y = 24$ x-intercept: $(3, 0)$; y-intercept: $(0, -4)$

76. $3x + 4y = 8$ x-intercept: $\left(\frac{8}{3}, 0\right)$; y-intercept: $(0, 2)$

Find the x-intercepts.

77. $y = x^2 + 8x + 15$ [5.4] $(-5, 0), (-3, 0)$

78. $y = 2x^2 - x - 3$ [5.5] $(-1, 0), \left(\frac{3}{2}, 0\right)$

Replace the blanks with constants to form a true equation. [8.1]

79. $x^2 - 14x + \underline{49} = (x - \underline{7})^2$

80. $x^2 + 7x + \underline{\frac{49}{4}} = \left(x + \underline{\frac{7}{2}}\right)^2$

SYNTHESIS

TW **81.** Before graphing a quadratic function, Cassandra always plots five points. First, she calculates and plots the coordinates of the vertex. Then she plots *four* more points after calculating *two* more ordered pairs. How is this possible?

TW **82.** If the graphs of $f(x) = a_1(x - h_1)^2 + k_1$ and $g(x) = a_2(x - h_2)^2 + k_2$ have the same shape, what, if anything, can you conclude about the a's, the h's, and the k's? Why?

Write an equation for a function having a graph with the same shape as the graph of $f(x) = \frac{3}{5}x^2$, but with the given point as the vertex.

83. $(4, 1)$ ⊡ **84.** $(2, 6)$ ⊡ **85.** $(3, -1)$ ⊡

86. $(5, -6)$ ⊡ **87.** $(-2, -5)$ ⊡ **88.** $(-4, -2)$ ⊡

For each of the following, write the equation of the parabola that has the shape of $f(x) = 2x^2$ or $g(x) = -2x^2$ and has a maximum value or a minimum value at the specified point.

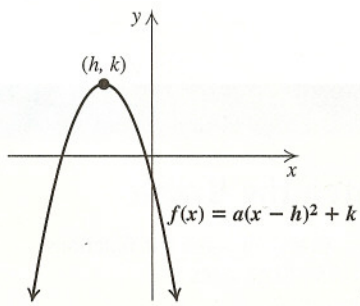
89. Minimum: $(2, 0)$ $f(x) = 2(x - 2)^2$

90. Minimum: $(-4, 0)$ $f(x) = 2(x + 4)^2$

91. Maximum: $(0, 3)$ $g(x) = -2x^2 + 3$

92. Maximum: $(3, 8)$ $g(x) = -2(x - 3)^2 + 8$

Use the following graph of $f(x) = a(x - h)^2 + k$ for Exercises 93–96.



93. Describe what will happen to the graph if h is increased. The graph will move to the right.

94. Describe what will happen to the graph if k is decreased. The graph will move down.

95. Describe what will happen to the graph if a is replaced with $-a$. The graph will be reflected across the x-axis.

96. Describe what will happen to the graph if $(x - h)$ is replaced with $(x + h)$. The graph will move to the right. The vertex will be $(-h, k)$.

Find an equation for a quadratic function F that satisfies the following conditions.

97. The graph of F is the same shape as the graph of f, where $f(x) = 3(x + 2)^2 + 7$, and $F(x)$ is a minimum at the same point that $g(x) = -2(x - 5)^2 + 1$ is a maximum. $F(x) = 3(x - 5)^2 + 1$

98. The graph of F is the same shape as the graph of f, where $f(x) = -\frac{1}{3}(x - 2)^2 + 7$, and $F(x)$ is a maximum at the same point that $g(x) = 2(x + 4)^2 - 6$ is a minimum. $F(x) = -\frac{1}{3}(x + 4)^2 - 6$

⊡ Answers to Exercises 67–71 and 83–88 are on p. IA-19.